Chapter 11 Problems

1. Imagine a particle confined to an infinitely high walled box, between $0 \le x \le a$. Assume that it is prepared in a coherent superposition state described by the time-dependent wavefunction

$$\psi(t) = c_1(t)\phi_1 + c_2(t)\phi_2$$

where the coefficients satisfy the equation $c_1(t)c_1^*(t) + c_2(t)c_2^*(t) = 1$, and the functions, ϕ_n, are the particle in a box eigenfunctions

$$\phi_n = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right) \quad \text{with } n = 1, 2, \ldots.$$

(a) Use the time-dependent Schrödinger equation (see Eq. (3.11)) to show that

$$c_m(t) = c_m(0)\exp(-i\varepsilon_m t/\hbar)$$

where ε_m is the energy of the eigenstate ϕ_m, with m = 1 or 2. $c_m(0)$ refers to the value coefficient $c_m(t)$ at t = 0.

(b) Obtain an expression for $\psi(t)\psi^*(t)$ in terms of ϕ_1 and ϕ_2.

(c) Compare the temporal behaviour of $\psi(t)\psi^*(t)$ when $c_1(0) = 1$, $c_2(0) = 0$ and $c_1(0) = c_2(0) = 1/\sqrt{2}$. Comment on the answers that you obtain.

[Hint: Based on the expression you obtained in part (b), work out $\psi(t)\psi^*(t)$ in terms of ϕ_1 and ϕ_2 at $t = 0$ and $t = \pi/\omega$, where $\omega = (\varepsilon_2 - \varepsilon_1)/\hbar$.]

2. (a) For a pulse with a Gaussian distribution of modes (FWHM 100 nm, centred at 800 nm) and Gaussian temporal intensity distribution, what is the minimum FWHM pulse duration that can be achieved?

(b) For a pulse of intensity $I = 10^{14}$ W cm^{-2}, calculate the refractive index of Ti:sapphire at 800 nm. Determine the phase delay at 800 nm after travelling through:

i. 5 mm of Ti:sapphire ($n_0 = 1.76$, $n_2 = 3.1 \times 10^{-16}$ cm^2 W^{-1});

ii. 5 mm BK7 glass ($n_0 = 1.51$, $n_2 = 3.5 \times 10^{-16}$ cm^2 W^{-1}).

3. Show that the autocorrelation function is given by $\langle\Psi(t)|\Psi(0)\rangle = \sum_v a_v^2 g_v^2 e^{iw_v t}$. Plot the modulus of the autocorrelation function following excitation of a

wavepacket created from vibrational states centred around $\bar{v}$ = 21, with a transform-limited Gaussian laser pulse of 40 fs duration, for

(a) a harmonic oscillator with ω_e = 170 cm^{-1};

(b) and anharmonic oscillator with ω_e = 170 cm^{-1} and $\omega_e x_e$ = 2 cm^{-1}.

For a Gaussian pulse, $g_v = \exp[-2\ln 2(E_\upsilon - E_{\bar{\upsilon}})^2/\mathrm{FWHM}^2]$. For simplicity, assume that a_v^2 is approximately constant.